

# Wave impact on elastically supported horizontal deck

W. Sulisz\*, P. Wilde, M. Wisniewski

*Polish Academy of Sciences, Institute of Hydroengineering, Kosciarska 7, 80953 Gdansk, Poland*

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## Abstract

Laboratory experiments supported by theoretical investigations were applied to study the wave impact on a horizontal deck, deck vibrations, and phenomena related to wave-induced vibrations. The analysis shows that one can distinguish four characteristic stages of deck vibrations between two consecutive wave impacts. In the first stage, the wave impact induces very high-frequency waves in the deck structure and in the wave flume. There are important components with acoustic frequencies. In the second stage, the deck vibrates in contact with water. The frequencies of vibrations in water are much lower than the corresponding frequencies of the free vibrations in air. The third stage of wave-induced vibrations is a transitional one. At the beginning of the third stage, the deck vibrates in contact with water and at the end of this stage it vibrates in air as a free rigid body. In this process water is shed from the deck. Sometimes the process of shedding is violent, which induces high-frequency vibrations with relatively large amplitudes. The fourth stage of wave-induced vibrations corresponds to the free vibrations of the deck in air. Similar four stages can be distinguished in the pressure records. Additional information regarding wave impact on the deck and the phenomena related to the impact can be obtained by the analysis of mechanical energy. The analysis of mechanical energy is especially helpful in understanding dissipation processes and phenomena in the transitional stage of wave-induced vibrations. In particular, the analysis of energy explains why the amplitudes of acceleration are increasing in the transitional stage of wave-induced vibrations.

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## 1. Introduction

Superstructures of breakwaters, decks of jetties or platforms, elements of balustrades, etc., are typical elements of maritime structures supported above still water level. The elements of maritime structures are often subject to wave attacks despite of precautions at the design stage. In the case of coastal structures the waves may reach the deck of a structure due to significant changes in bathymetry. In the case of offshore structures, especially offshore platforms, waves may reach the decks of structures due to its settlement. However, most often wave crests reach decks or other elements of maritime structures supported above still water level when the structure is exposed to the attack of extreme waves or extreme wave events. Wave impact loads and wave-induced vibrations of coastal and offshore structures often lead to partial or permanent damage of structures, which requires investigation.

The studies conducted on wave action on a plate located close to the still water level refer mainly to decks of platforms or jetties and focus on impact forces. Simplified approaches to assess uplift forces were proposed by Wang

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\*Corresponding author. Tel.: +48 58 5522011; fax: +48 58 5524211.

*E-mail address:* sulisz@ibwpan.gda.pl (W. Sulisz).

(1970), El-Ghamry (1971), French (1979), Shih and Anastasiou (1992), Isaacson and Bhat (1996), Bea et al. (1999) and McConnell et al. (2003). These approaches were based on momentum and energy considerations or semi-analytical formulas. Kaplan et al. (1995) proposed a set of equations to calculate wave forces on a platform, which are often applied in design. Numerical modelling of wave action on a horizontal deck have been conducted by Lai and Lee (1987), Baarholm and Faltinsen (2004) and Ren and Wang (2004). The numerical modelling is based on the application of a finite element method, a boundary element method, and a volume of fluid method.

Laboratory investigations and numerical modelling of wave action on a plate located close to the still water level have focused on the prediction of impact forces. The problem of the vibration of the plate was given limited attention. However, the vibrations should be included in the analysis of wave impact on a deck because they increase microcracks on a structure surface. Monitoring of structures exposed to wave impact, including concrete and steel structures, indicates that the failure of a structure is often not caused directly by wave impact but it is rather a result of long process involving the cumulation of cracks. Thus, a comprehensive analysis of wave impact on a deck must include an analysis of impact forces as well as wave-induced vibrations.

An original approach to study the problem of wave impact and wave-induced deck vibrations was applied by Wilde et al. (1998). They investigated a standing wave action on a horizontal deck. The deck was elastically supported to study combined effects of wave impact and wave-induced vibrations. The results showed several interesting phenomena, including stages of extremely high accelerations. The magnitude of wave-induced acceleration was around  $10g$ , which motivated the present work on the progressive wave impact on a deck and wave-induced vibrations.

In this work, laboratory experiments, supported by theoretical investigations, are applied to study the wave impact on a horizontal deck, deck vibrations, and phenomena related to wave-induced vibrations. First, laboratory experiments are conducted to study free vibrations of the deck in air and in contact with water. The analysis of experimental data is supported by theoretical investigations to determine parameters of free vibrations. Then, wave impact on the deck is investigated by conducting laboratory experiments in a wave flume. Experimental data are analysed with emphasis on wave impact and wave-induced vibrations. Finally, the problem of pressure and mechanical energy is addressed and conclusions are reached.

## 2. Description of experiments

Laboratory experiments were performed in the wave flume of the Institute of Hydroengineering in Gdansk. The wave flume is 64 m long, 0.6 m wide and 1.4 m deep and it is equipped with a programmable wavemaker. The model of the deck was constructed in the form of a box shown schematically in Fig. 1.

The model of the deck was made from plexiglas plates stiffened by ribs, so that it can be considered as a rigid body. The deck was supported by prestressed strings and springs. In the construction of the model, attention was paid to decrease damping as much as possible. As a consequence, the effect of damping on the frequencies of free vibrations was very small, practically negligible.

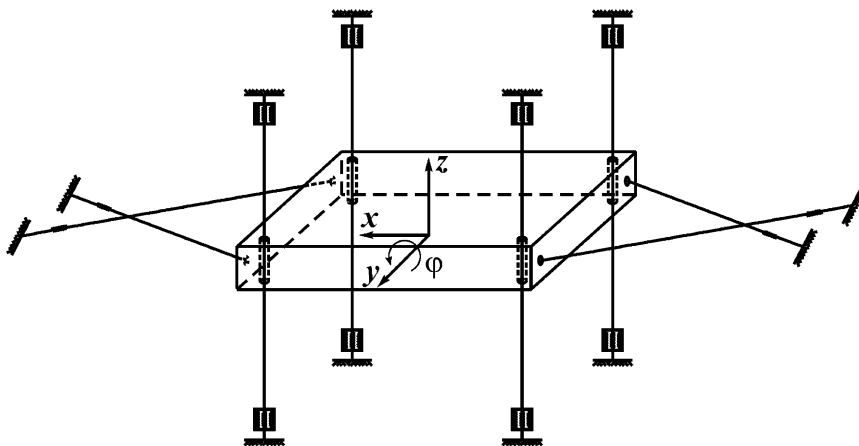


Fig. 1. Sketch of the deck model and coordinate system.

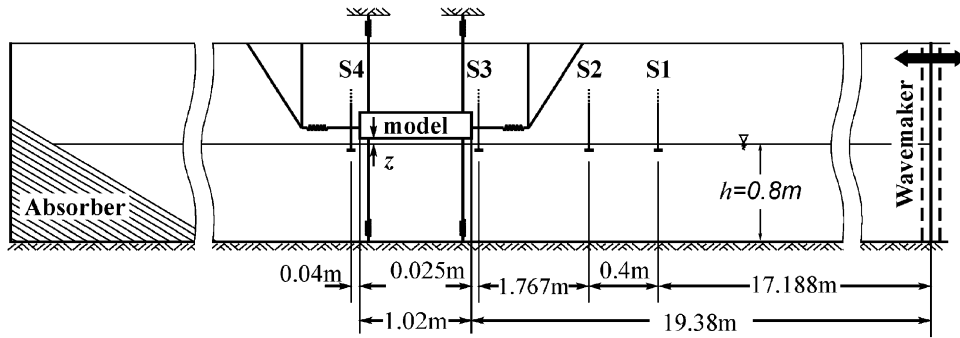


Fig. 2. The model of the deck in the wave flume.

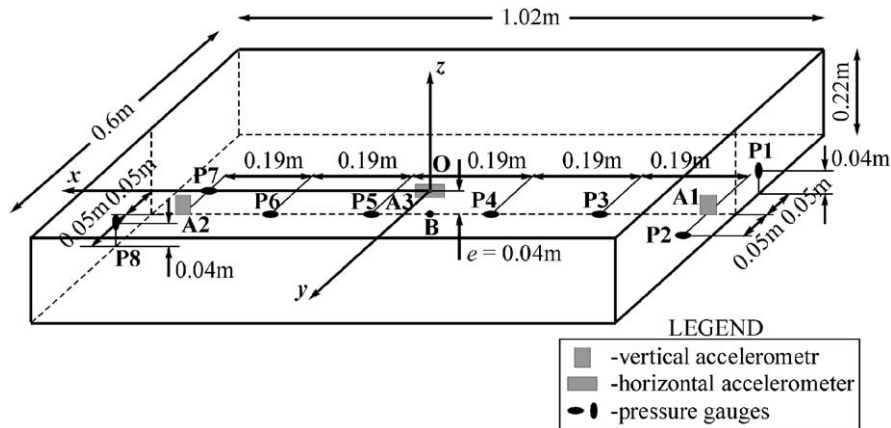


Fig. 3. Arrangement of pressure gauges and accelerometers.

The model of the deck was installed in the wave flume as it is shown schematically in Fig. 2. The measured weight of the model was  $m = 19.49$  kg, the measured spring constants were  $k_x = 56\,407.5$  N/m,  $k_z = 324\,711$  N/m and the calculated rotational constant,  $k_\phi = 83\,302.1$  Nm. The moment of inertia was calculated on the basis of measured natural frequency and the calculated value of the rotational spring constant. The origin of the local Cartesian coordinate system is located at  $e = 0.04$  m above the bottom of the deck. It corresponds to the center of mass of the rectangular box.

The motion of the deck in this study was restricted to 2-D oscillations. Four wave gauges, denoted by S1, S2, S3, and S4, were installed in the wave flume to measure free-surface elevations. The positions of the wave gauges are shown in Fig. 2. Moreover, three accelerometers and eight pressure gauges were installed in the deck to measure its accelerations and wave-induced pressures. The positions of vertical accelerometers, denoted by A<sub>1</sub> and A<sub>2</sub>, and the horizontal accelerometer, denoted by A<sub>3</sub>, are shown in Fig. 3. The pressure gauges, denoted by P<sub>2</sub>, ..., P<sub>7</sub>, were installed in the bottom of the deck, and two gauges, P1 and P8, were installed on the side walls.

The deck was exposed to the attack of wave trains composed of regular and irregular waves. The investigations of wave-induced vibrations preceded the study of the free vibration of the deck in air and in contact with water. A detailed description of all tests is summarised in Sobierajski and Wisniewski (2004).

### 3. Analysis of experimental data

#### 3.1. Description of vibrations

A structure undergoing 2-D vibrations can be considered to have three degrees of freedom, corresponding to displacements along the  $x$ - and  $z$ -axes,  $u(t)$  and  $w(t)$ , and rotation around the  $y$ -axis,  $\phi(t)$ , (three generalised

displacements). In general, the vibration of such a structure is governed by a set of three coupled differential equations with three unknown generalised displacements. Accordingly, the vibrations of the deck can be described by the set of the following equations:

$$\begin{aligned}
 m_x \ddot{u} + c_x \dot{u} + k_x u &= F_x, \\
 m_z \ddot{w} + c_z \dot{w} + k_z w &= F_y, \\
 J \ddot{\varphi} + c_\varphi \dot{\varphi} + k_\varphi \varphi &= M,
 \end{aligned}
 \tag{1}$$

where  $F_x$ ,  $F_y$  and  $M$  are generalised external forces.

Further simplifications in the description of the deck vibrations are possible because the model of the deck has a plane of symmetry and the center of mass of the system is located in the plane of symmetry. Moreover, the amplitudes of the vibrations and damping are small.

### 3.2. Free vibrations

First, the free vibrations of the deck installed in an empty wave flume were analysed. The vibrations were initiated by a sudden release of initial displacement. An example of measured accelerations in air, the amplitudes of corresponding Fourier series, and the zoomed section of the initial stage of motion are shown in Fig. 4.

The plots in Fig. 4 show vibrations of relatively high frequencies in the initial stage of motion, followed by typical damped vibrations. The vibrations of high frequencies are induced by the sudden release of initial displacement. Then, the deck vibrates with frequencies corresponding to the free vibrations of a rigid body. The results presented in Fig. 4 show that, in the second stage of motion, vertical and rotational modes of vibrations dominate.

It is assumed that the acceleration corresponding to the stage of damped vibrations is described by the following formula

$$\ddot{U}(t) = \Re e[(a + ib) \exp[(-\eta - i2\pi f)t]],
 \tag{2}$$

where  $a$  and  $b$  are amplitudes of acceleration,  $f$  is the frequency, and  $\eta$  is the damping coefficient. The complex form is convenient because the integration of exponential function is simple. For example, the corresponding velocities and

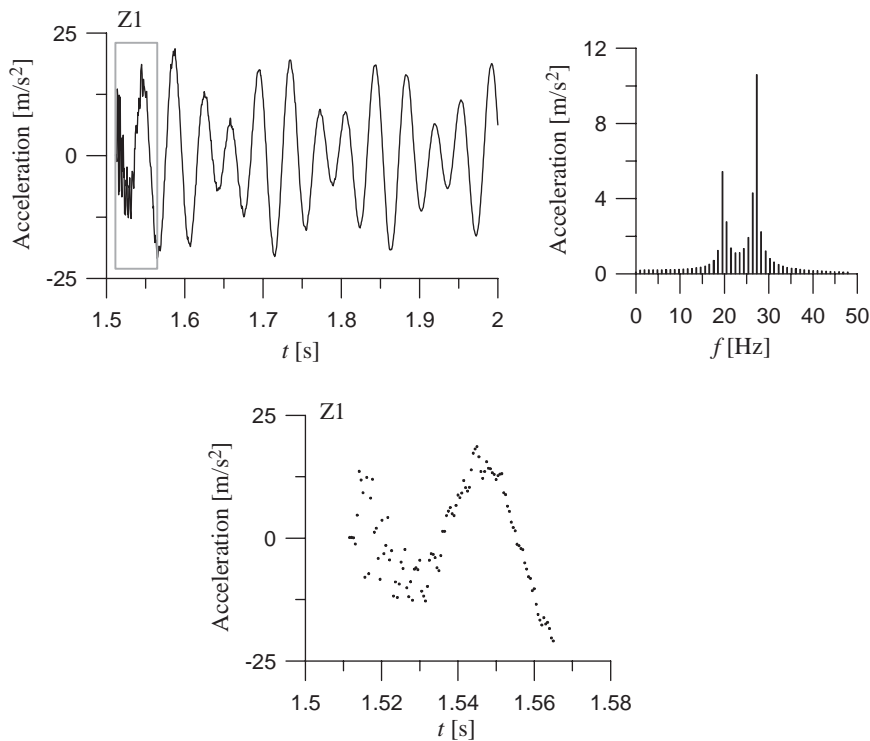


Fig. 4. Record of vertical acceleration of the deck and the amplitudes of corresponding Fourier series.

displacements are

$$\begin{aligned} \dot{U}(t) &= \Re e \left[ \frac{a + ib}{(-\eta - i2\pi f)} \exp[(-\eta - i2\pi f)t] \right], \\ U(t) &= \Re e \left[ \frac{a + ib}{(-\eta - i2\pi f)^2} \exp[(-\eta - i2\pi f)t] \right]. \end{aligned} \tag{3}$$

Eqs. (2) and (3) are applied to determine the frequency and the damping coefficients from the records of the free vibrations of the deck in air. The results of the analysis of experimental data are presented in Table 1.

The frequency of the free vibrations of the deck in air can also be predicted theoretically. Namely, one can assume that the motion of the deck is described by a set of uncoupled equations (1) with generalised external forces equal to zero. Such an approach leads to a standard problem for eigenfrequencies and eigenmodes typically used in the analysis of the vibration of structures. The predicted frequencies are presented in Table 1.

The differences between calculated and measured frequencies are likely due to simplifications applied in the theoretical analysis and experimental procedure. Namely, in the calculations only the masses of important elements of the system were taken into account. Moreover, the values of elastic coefficients were determined in simple laboratory experiments. These are likely the main reasons of the observed differences between calculated and measured frequencies.

Next, the free vibrations of the deck in contact with water at different drafts were analysed. Different drafts were achieved by increasing the water depth in the wave flume. The investigations of the free vibrations were included in the study because it was found that the natural frequencies drastically decrease when the deck is in contact with water in comparison with corresponding natural frequencies of the vibrations of the deck in air (Sulisz et al., 2004a, b). Moreover, the study of the free vibrations of the deck in contact with water are expected to be helpful in understanding the complex phenomena accompanying the wave impact on the deck and wave-induced deck vibrations.

In the case of the free vibrations of the deck in contact with water, the center of mass moves to a new position. However, the change of the position is very small, because the added mass of water in horizontal oscillations is very small. The analysis of experimental data of the free vibrations of the model in contact with water show that the effect of added mass of water on the horizontal frequency  $f_x$  is small, as expected. In order to assess the parameters of free vibrations of the deck in contact with water, it is reasonable to assume that the motion of the deck is governed by the set of uncoupled equations (1). Moreover, because the damping in experiments was very small, so the effect of damping on the values of the frequencies is negligible, one can apply following:

$$m_x = k_x/\omega_x^2, \quad m_z = k_z/\omega_z^2, \quad J = k_\phi/\omega_\phi^2, \tag{4}$$

where  $\omega = 2\pi f$ . Moreover, it is convenient to introduce following:

$$c_x = 2\eta_x m_x, \quad c_z = 2\eta_z m_z, \quad c_\phi = 2\eta_\phi J. \tag{5}$$

Eq. (4) are applied to assess added masses for horizontal and vertical modes of vibration and added moment of inertia for rotational vibrations. The experimental values of natural frequencies, the damping coefficients in contact with water, and the calculated values of masses and moments of inertia as functions of the draft are presented in Table 2.

The comparison of the natural frequencies corresponding to the vibrations of the deck in air and in contact with water shows that for the vertical and the rotational modes the frequencies drastically decrease due to the interaction of the deck with water. The effect of the interaction of the deck with water on the parameters of the horizontal mode is very limited. The changes in the magnitude of the natural frequencies due to the contact of the deck with water were expected (Blevins, 1977), however not in such a drastic form. The results also show that the vibrating mass  $m_z$  substantially increases in contact with water. For example, for  $d = 0.01$  m, the added mass increases from 19.49 to 267.30 kg. It means that the covibrating mass of fluid is 247.81 kg for a draft  $d$  equal to 0.01 m. The added mass increases when the draft increases. A similar tendency is exhibited by the added moment of inertia, but there is no significant increase of added mass in horizontal vibrations when the contact with fluid is initiated.

Table 1  
Parameters of free vibrations of the deck in air

Mode	Mass, inertia	Elastic coefficient $k$	Calculated frequency $f$	Measured frequency $f$	Damping coefficient $\eta$
$x$	19.49 kg	56 407 N/m	8.56 Hz	8.55 Hz	0.308 s <sup>-1</sup>
$z$	19.49 kg	32 4711 N/m	20.54 Hz	19.82 Hz	0.339 s <sup>-1</sup>
$\phi$	2.88 kg m <sup>2</sup>	83 302 Nm	27.07 Hz	27.07 Hz	0.265 s <sup>-1</sup>

Table 2  
Parameters of free vibrations of the deck in contact with water

Draft $d$	0.01 m	0.03 m	0.06 m	0.09 m	0.12 m
$f_x$ (Hz)	8.45	8.36	8.12	7.75	7.36
$f_z$ (Hz)	5.55	5.20	4.92	4.74	4.62
$f_\phi$ (Hz)	13.16	12.7	12.39	12.21	12.11
$\eta_x$ (l/s)	0.39	0.40	0.65	0.64	0.83
$\eta_z$ (l/s)	1.15	0.62	1.01	0.72	0.55
$\eta_\phi$ (l/s)	0.90	0.53	0.47	0.54	0.60
$m_x$ (kg)	19.99	20.45	21.66	23.77	26.36
$m_z$ (kg)	267.3	304.03	339.2	365.99	384.88
$J$ (kg m <sup>2</sup> )	12.18	13.15	13.74	14.14	14.40

### 3.3. Vibrations due to wave impact

In order to study the vibrations of the deck due to wave attack, free-surface elevations were measured before the deck was installed in the wave flume. Fig. 5 shows time series of the free-surface elevations recorded by gauge S3 and S4 with the sampling rate equal to 200 Hz. The plots in Fig. 5 also show the amplitudes of the corresponding Fourier series. The basic wave of frequency  $f_1 = 0.2783$  Hz is accompanied by higher-order bound Stokes waves (Sulisz and Hudspeth, 1993). The free-surface elevation may be described by the standard Stokes expression

$$\zeta(x, t) = \sum_{n=1} [a_n \cos n(k_1 x - \omega_1 t) + b_n \sin n(k_1 x - \omega_1 t)], \quad (6)$$

where  $k_1$  is the wave number and  $\omega_1$  is the angular wave frequency related to the wave frequency by the formula,  $\omega_1 = 2\pi f_1$ .

The deck was installed in the wave flume after the measurements of incident wave profiles. The bottom of the deck was located at different position above the still water level (Sobierajski and Wisniewski, 2004). The results presented in this paper refer to the deck installed 0.04 m above the still water level. The free-surface elevations were recorded with sampling rate equal to 5000 Hz. The recording was limited to few wave periods. Fig. 6 shows time series of the free-surface elevations recorded by gauge S3 and S4, and the amplitudes of corresponding Fourier series.

In the attack of regular waves on the deck, one can distinguish four characteristic stages of wave-induced vibrations between two consecutive wave impacts. A vertical acceleration recorded between two consecutive wave impacts is plotted in Fig. 7. The plot clearly shows four characteristic stages of wave-induced vibrations.

The record of vertical acceleration corresponding to the first stage of wave-induced vibrations, Z1, is zoomed in Fig. 8. The plots in Fig. 8 show that, in the first stage of wave impact, the process of vibrations is very complex and there are many components with very high frequencies. These components are due to the impulse that induces very high-frequency waves in the deck structure, in the water layer, and probably also in the frame of the wave flume. There are important components with acoustic frequencies. This may imply a need to include water compressibility in the modelling of this stage of wave impact. The modelling may also have to take into account the higher natural frequencies of the deck structure, e.g. plexiglas plates, ribs, etc. (Wilde et al., 1998).

The record of acceleration corresponding to the second stage of wave-induced vibrations, Z2, is zoomed in Fig. 9. The plots show recorded time series and the amplitudes of corresponding Fourier series.

The plots in Fig. 9 show that the second stage of wave-induced vibrations is far simpler than the initial stage of wave-induced vibrations. The results show that in the recorded time series there are two main components corresponding to frequencies  $f = 4.135$  and  $12.405$  Hz. A comparison with the parameters of vibrations presented in Table 2 indicates that these two components correspond to the frequencies,  $f_z$  and  $f_\phi$ , of the free rigid-body vibrations in contact with water.

The third stage of wave-induced vibrations, Z3, is a transitional one. At the beginning of the third stage the deck vibrates in contact with water and at the end of this stage it vibrates in air as a free rigid body—stage Z4. In this process water is shed from the deck. Sometimes the process of shedding is violent, which induces high-frequency vibrations with relatively large amplitudes. A transition from the vibrations in contact with water to the free vibration in air is a complex process that requires an individual approach and a nonstandard analysis.

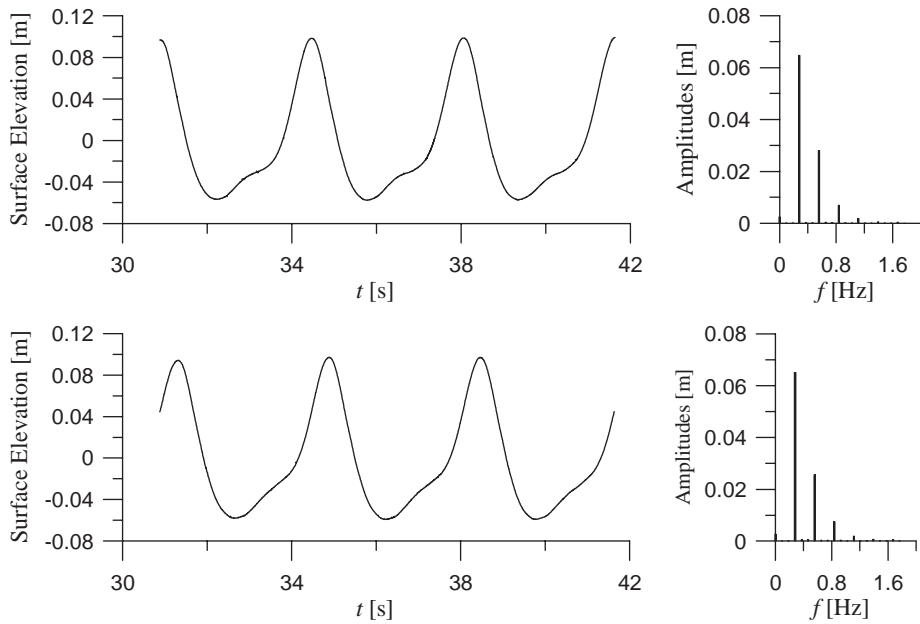


Fig. 5. Free-surface elevation recorded by gauge S3 and S4, and the amplitudes of corresponding Fourier series.

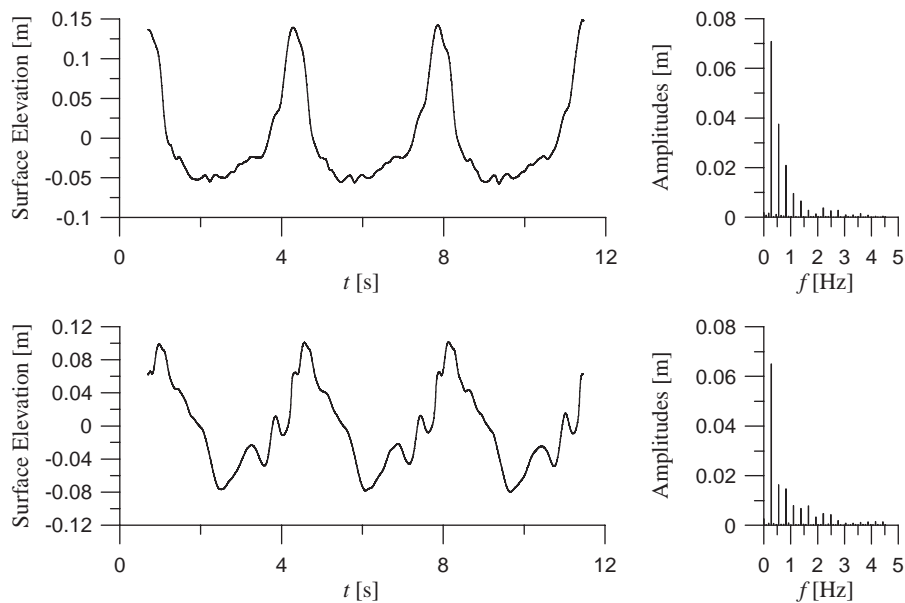


Fig. 6. Record of free-surface elevation from gauge S3 and S4, and the amplitudes of corresponding Fourier series.

In the present analysis the acceleration corresponding to the vertical and rotational modes of vibration were determined from the accelerations measured by gauges A1 and A2, which basically contain only these two components of vibrations. The vertical component of the deck acceleration  $\ddot{w}(t)$  was calculated as a mean of the accelerations recorded by gauge A1 and A2, and the angular acceleration was calculated from the difference of the records divided by the distance between these two gauges. The horizontal acceleration  $\ddot{i}(t)$  was estimated on the basis of the measurements from the accelerometer A3. High-frequency vibrations induced by the violent shedding of water were filtered out, which is illustrated in Fig. 10. Then, the standard procedure was applied to find “up-crossing” and “down-crossing” in each

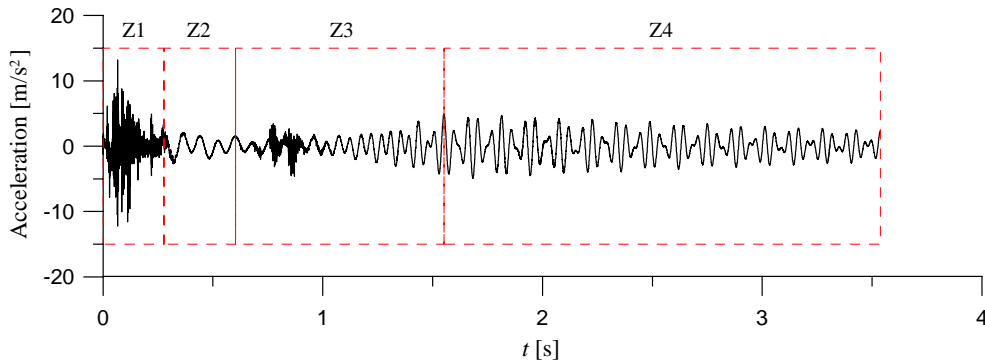


Fig. 7. Record of vertical acceleration between two consecutive wave impacts.

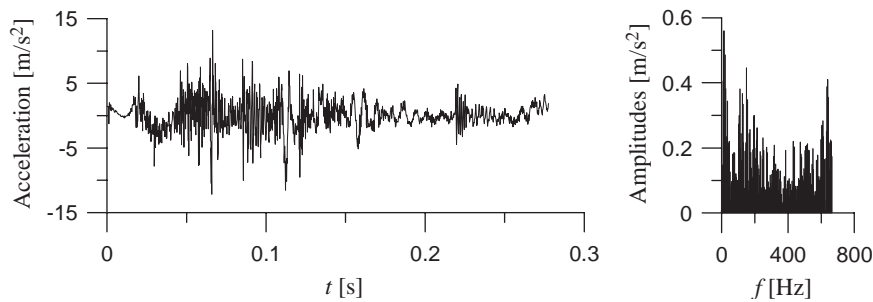


Fig. 8. Record of vertical acceleration and the amplitudes of corresponding Fourier series for section Z1.

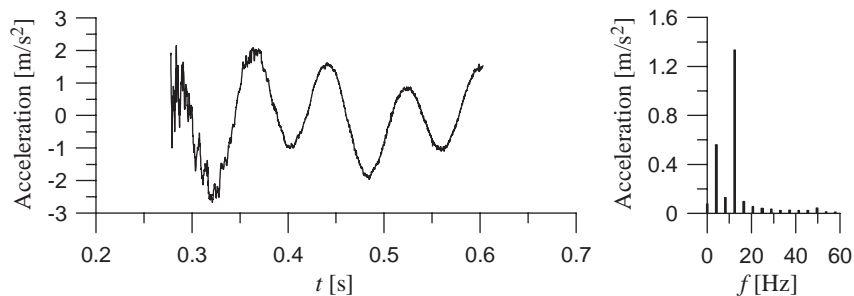


Fig. 9. Record of vertical acceleration and the amplitudes of corresponding Fourier series for section Z2.

interval. The record in each interval was approximated by a sine curve, which provided the frequency. Fig. 11 shows how the frequencies change in the transition from the vibrations of the deck in contact with water to the free vibrations in air.

The fourth stage of wave-induced vibrations corresponds to the free vibrations of the deck in air. The results corresponding to the zoomed section Z4 are shown in Fig. 12.

The plots show that in the recorded time series there are two basic components corresponding to the frequencies  $f = 18.978$  and  $26.461$  Hz. These components correspond to the frequencies  $f_z$  and  $f_\phi$  of the free vibration of the deck in air (Table 1). The analysis of data in this interval can be conducted by applying relations (2) and (3).

One should realize that the wave impact on the deck affects the wave field in the wave flume and changes the wave energy spectrum. The Stokes formula, Eq. (6), cannot be applied to describe the free-surface elevation in the wave flume after the wave impact. The measured free-surface elevation is the superposition of waves initiating the vibrations of the deck, reflected waves, etc. The detailed analysis indicates that in the recorded wave profile there are no significant



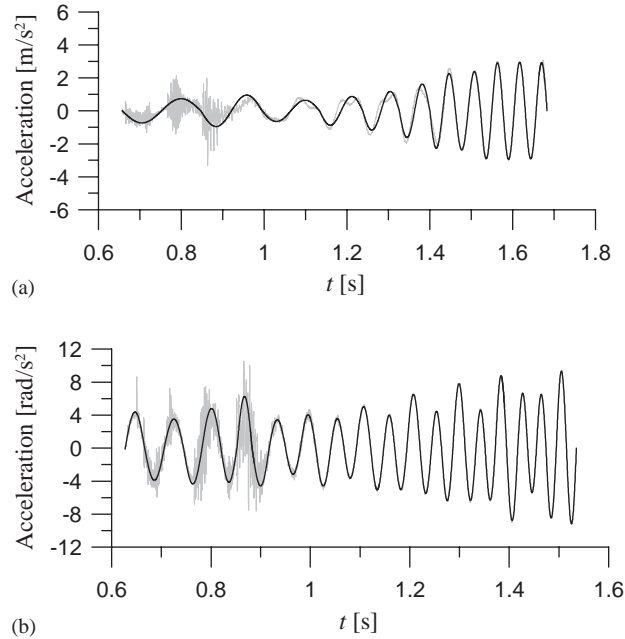


Fig. 10. Raw and approximated values of (a) the vertical and (b) rotational accelerations.

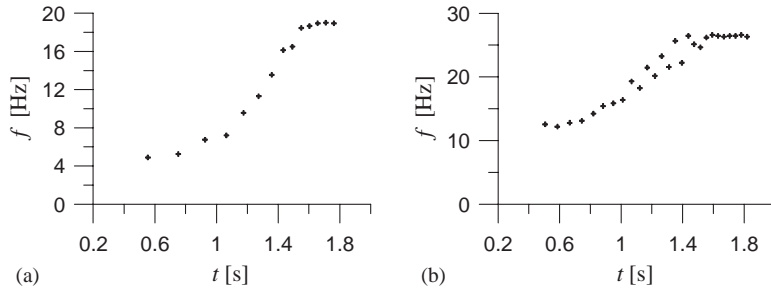


Fig. 11. Discrete values of the frequencies for (a) vertical and (b) rotational modes.

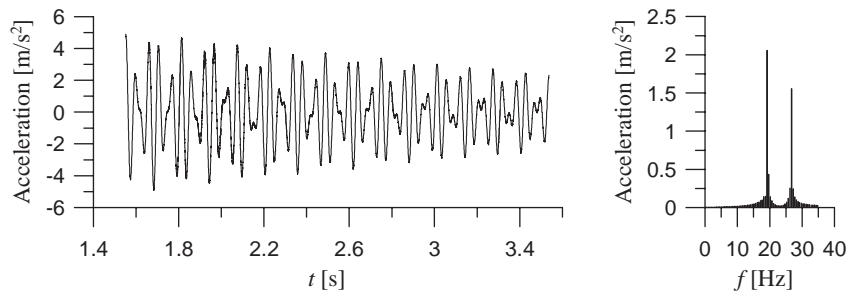


Fig. 12. Record of vertical acceleration and the amplitudes of corresponding Fourier series for section Z4.

components corresponding to the natural frequencies of the vibrating deck. Thus, in the modelling of the wave field it is reasonable to assume that the deck is a rigid and stationary structure. Such a model may be described by applying a nonlinear wave theory with mixed boundary conditions at the free surface (Sulisz, 1993, 1999).

### 3.4. Pressure due to wave impact

The pressures measured by the gauges fixed in the bottom of the deck provide valuable insight into the physics of wave impact on the deck. Fig. 13 shows the pressure recorded by the gauge P7.

There are four characteristic stages in the pressure record, which correspond to the described stages of wave-induced vibrations. In the first stage, a rapid increase and then a decrease of pressures is observed. This stage corresponds to the first stage of wave impact on the deck. In the second stage the pressure oscillates with frequency corresponding to the frequencies of free vibrations of the deck in contact with water. The third stage is a transitional one. In the fourth stage the deck vibrates in air and the dynamic pressure is zero.

In order to establish the division of pressure record into four stages it is necessary to introduce some approximations. In the impact stage, the rapid increase of pressure can be approximated by a straight line. The attenuation of pressure is slower and can be approximated by an exponential function. The values of the coefficients for a straight line and an exponential function were calculated by the least-squares method. Fig. 14 shows the raw data and the approximated pressure in the impact stage, and Fig. 15 presents the differences between the measured and approximated pressure, as well as the corresponding amplitudes of Fourier series.

The analysis of the pressure recorded at the second stage of the deck vibrations was conducted by applying the Kalman filters (Wilde and Kozakiewicz, 1993). In the applied decomposition the linear mathematical models of random functions with dominant frequencies were used. As the dominant frequencies, the frequencies of 6 and 10 Hz were used as representative for the vertical and rotational modes of motion of the deck in contact with water, and the frequencies 1 and 22 Hz were applied as representative for the lower and higher frequencies. The highest frequencies appearing in the rest correspond to the differences between the raw data and the sum of the estimated functions. The estimated functions and the corresponding amplitudes of Fourier series are shown in Fig. 16(a)–(c).

The amplitudes of the Fourier components presented on the first graph, Fig. 16(a), show that there is an important low-frequency component. This component of pressure, corresponding to 1 Hz, is the result of the interaction of a wave crest with the deck. The bottom of the deck was located 0.04 m above the still water level, so the deck was exposed only to the action of wave crests, which was limited to about 1 s for each wave. The second graph, Fig. 16(b), shows the important components with the frequencies corresponding to the frequencies of the vertical vibrations of the deck. The

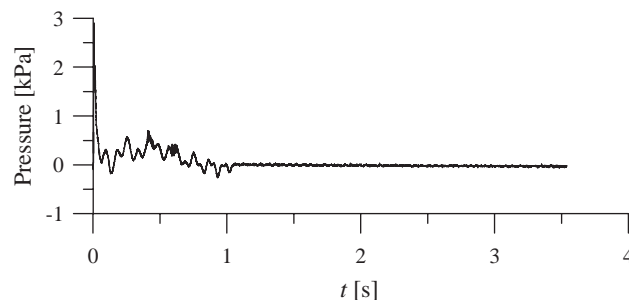


Fig. 13. Record of pressure at the bottom of the deck.

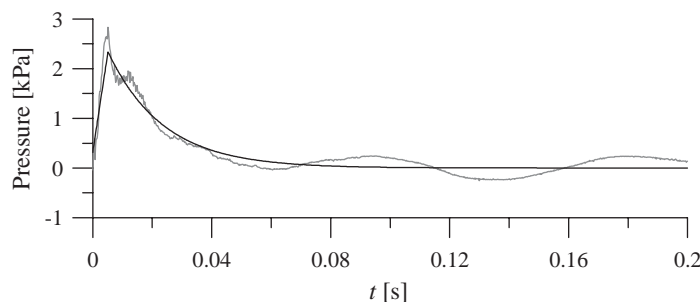


Fig. 14. Raw and approximated values of pressure within the impact stage.

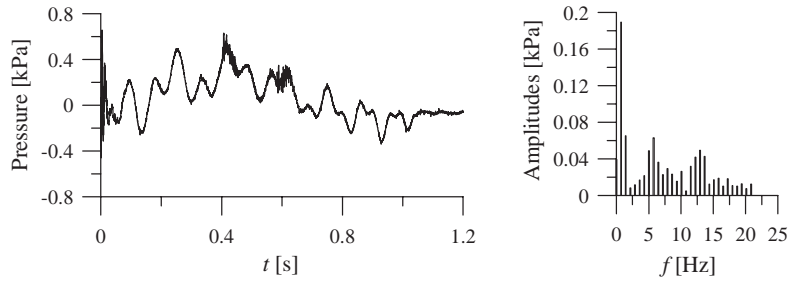


Fig. 15. The difference between the measured and approximated impact pressures.

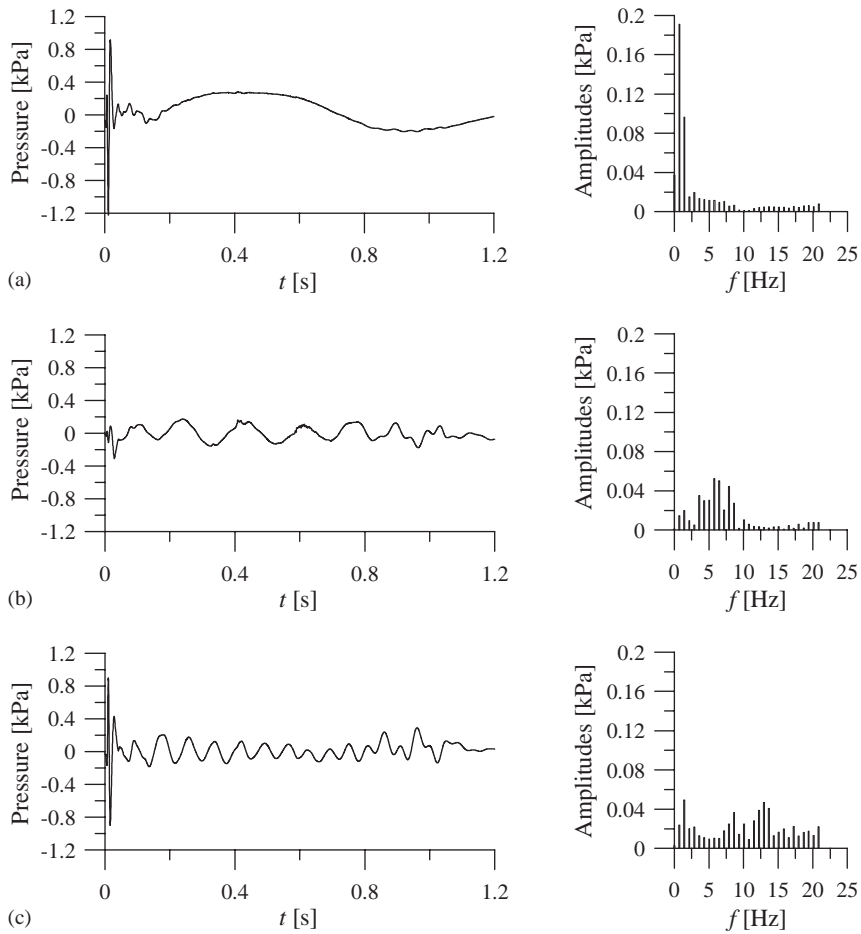


Fig. 16. Components of Kalman approximation corresponding to dominant frequency (a) 1 Hz, (b) 6 Hz, (c) 10 Hz.

third graph, Fig. 16(c), shows the components corresponding to the rotational mode of vibrations. The error of the Kalman approximation is about 2.3%.

The plots in Fig. 16 indicate a complex pressure behaviour at the second stage of the deck vibrations. It should be noted that in the pressure records the frequency components corresponding to the free vibrations of the deck in contact with water are by far more pronounced than in the records of free surface where these components are negligible.

The third stage is a transitional one. The pressure records show that the process of water shedding does not influence the pressure so much as in the case of accelerations. In the fourth stage the deck vibrates in air and the dynamic pressure is zero.

The measured pressure provides additional information regarding the wave impact on the deck. The relations between recorded accelerations and pressures are complex and are difficult to be predicted theoretically.

#### 4. Mechanical energy

The performed analysis of experimental data provides insight into the physics of wave impact on the deck. More information regarding this phenomenon can be obtained by the analysis of mechanical energy. The energy analysis is especially helpful in understanding dissipation processes and phenomena in the transitional stage of wave-induced vibrations. In particular, the analysis of energy may explain why the amplitudes of acceleration are increasing in the transitional stage Z3 of wave-induced vibration, which is clearly shown in the plots of Figs. 7 and 10.

The kinetic energy  $K(t)$  and the potential energy  $V(t)$  are estimated by the following relations:

$$\begin{aligned} 2K(t) &= m_x \dot{u}(t)\dot{u}(t) + m_z \dot{w}(t)\dot{w}(t) + J\dot{\varphi}(t)\dot{\varphi}(t), \\ 2V(t) &= k_x u(t)u(t) + k_z w(t)w(t) + k_\varphi \varphi(t)\varphi(t). \end{aligned} \quad (7)$$

Eqs. (7) can be applied to calculate the mechanical energy  $E(t)$  which is the sum of the kinetic energy  $K(t)$  and the potential energy  $V(t)$ , namely  $E(t) = K(t) + V(t)$ . The mechanical energy corresponding to the selected mode of vibration can also be calculated from the following approximate formula:

$$E = \frac{1}{2} k [\text{abs}(U)]^2 = \frac{1}{2} k \frac{a^2 + b^2}{(2\pi f)^4} \exp(-2\eta t), \quad (8)$$

where  $k$  is the spring constant corresponding to the selected mode of vibrations.

Eqs. (7) and (8) are applied to analyse the mechanical energy at the different stages of wave-induced deck vibration. The plots of the mechanical energy for the vibration of the deck in air (Z4) possess a standard form of damped vibrations. Time histories of the accelerations, velocities and displacements during the vibration stage of the deck in air are shown in Fig. 17, and Fig. 18 shows the kinetic and potential energies during this stage. The preceding transition from the vibrations in contact with water to free vibration in air is a complex process. The mechanical energy during

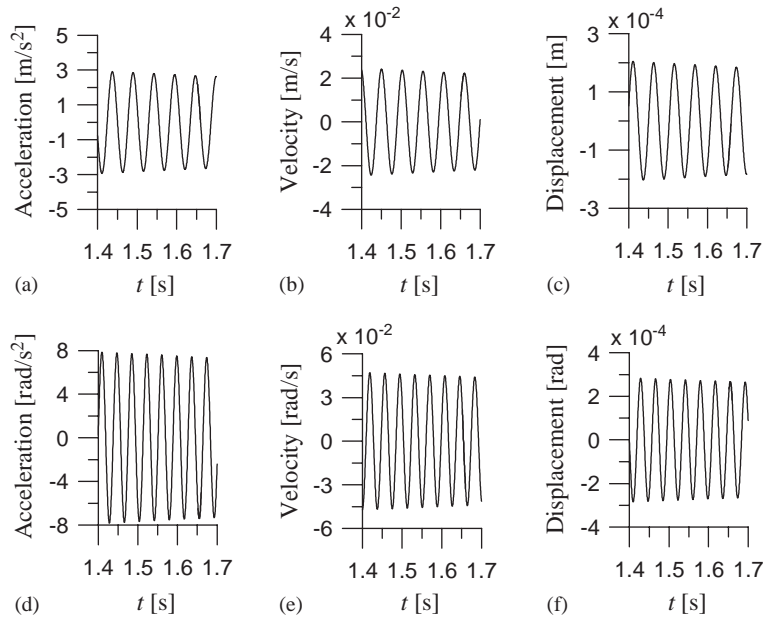


Fig. 17. Accelerations, velocities and displacements for (a–c) vertical and (d–f) rotational modes corresponding to the vibration stage Z4.

that stage can be calculated by applying Eq. (8), which is very convenient for assessing the mechanical energy because it is based on the values of the spring constants. The results are plotted in Fig. 19.

The plots in Fig. 19 are helpful in understanding the dissipation processes and some phenomena in the transitional stage of wave-induced vibrations. In particular, the results in Fig. 19 show that the energy decays, despite the increase of

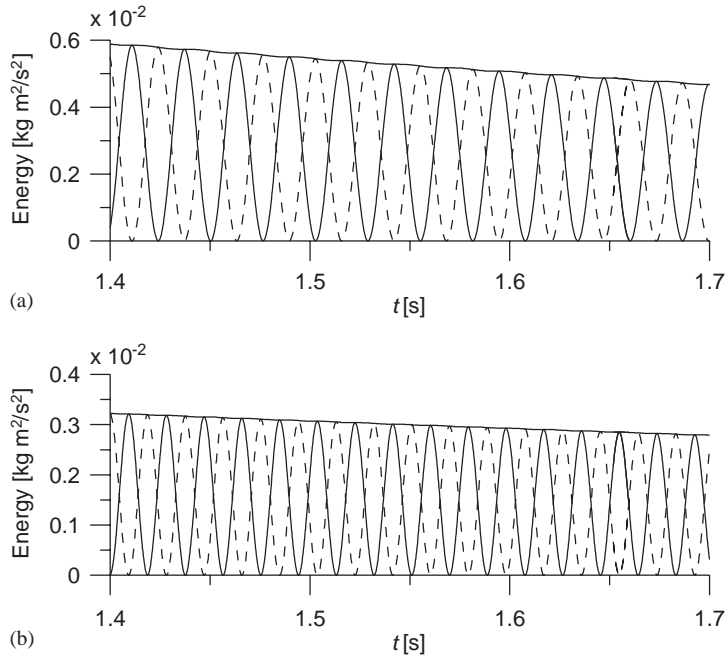


Fig. 18. Calculated kinetic (- - -), potential (—) and mechanical energy for (a) the vertical and (b) rotational mode (stage Z4).

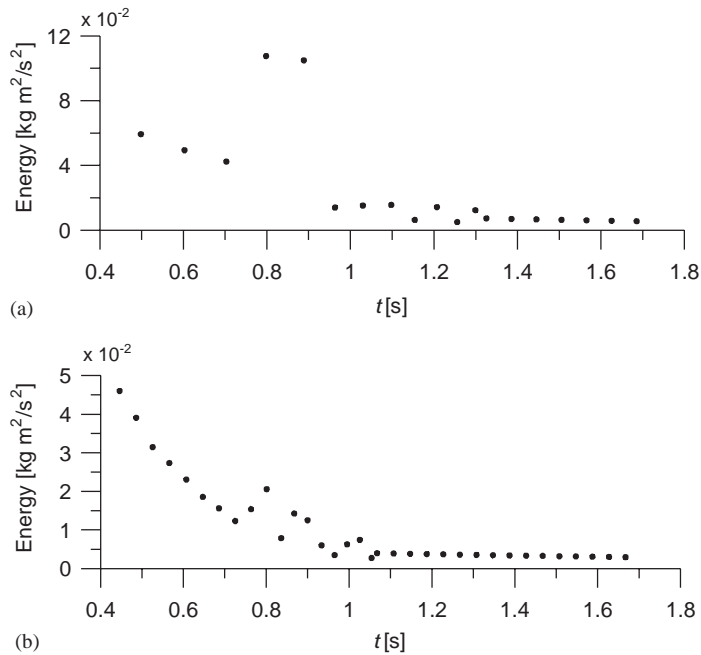


Fig. 19. Mechanical energy for (a) the vertical and (b) rotational mode (stage Z3).

the amplitudes of acceleration (Figs. 7 and 10). A detailed analysis of basic factors affecting the mechanical energy indicate that this is related to a significant effect of frequency on the magnitude of the mechanical energy. Eq. (8) shows that the mechanical energy is inversely proportional to the frequency to the power 4. This explains why the energy decays despite the increase of the amplitudes of acceleration in the transitional stage of wave-induced vibrations. In fact, Eq. (8) is very useful in the calculations of the mechanical energy because it is based on the values of the spring constants, and the spring constants are the same in the vibrations of the deck in air and in contact with water.

## 5. Conclusions

Wave-induced vibrations of coastal and offshore structures often lead to partial or total damage of structures and require detailed investigations. Laboratory experiments supported by theoretical investigations were applied to study wave-induced vibrations of a horizontal deck. Experimental data were analysed with emphasis on wave impact and wave-induced vibrations. The analysis was supported by theoretical investigations to determine parameters governing the vibrations.

The analysis shows that wave impact on the deck and wave-induced vibrations are very complex processes. In the attack of regular waves, one can distinguish four characteristic stages between two consecutive wave impacts. In the first stage of wave impact the impulse induces very high-frequency waves in the deck structure, in the water layer, and probably in the frame of a wave flume. There are important components with acoustic frequencies. The second stage of wave-induced vibrations is far simpler than the beginning of wave impact. The deck vibrates in contact with water. The frequencies of vibrations are much lower than the corresponding frequencies of vibrations in air. The third stage of wave-induced vibrations is a transitional one. At the beginning of the third stage the deck vibrates in contact with water and at the end of this stage it vibrates in air as a free rigid body. In this process, water is shed from the deck. Sometimes the process of shedding is violent which induces high-frequency vibrations with relatively large amplitudes. The fourth stage of wave-induced vibrations corresponds to the free vibrations of the deck in air.

The analysis of pressure also shows four characteristic stages in the pressure record. In the first stage, a rapid increase and then a decrease of pressures is observed. This stage corresponds to the first stage of wave impact on the deck. In the second stage, the pressure oscillates with frequencies corresponding to the frequencies of free vibrations of the deck in contact with water. The third stage is a transitional one. In the fourth stage, the deck vibrates in air and the dynamic pressure is zero.

Additional information regarding wave impact on the deck and the phenomena related to the impact can be obtained by the analysis of mechanical energy. The analysis of mechanical energy is especially helpful in understanding dissipation processes and phenomena in the transitional stage of wave-induced vibrations. In particular, the analysis of energy explains why the amplitudes of acceleration are increasing in the transitional stage of wave-induced vibration.

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